Academic Aims for Microeconomics C:

The course aims at giving the student knowledge of game theory, non-cooperative as well as cooperative, and its applications in economic models.
The student who successfully completed the course will learn the basic game theory and will be enabled to work further with advanced game theory. The student will also learn how economic problems, involving strategic situations, can be modeled using game theory, as well as how these models are solved. The course intention is thus, that the student through this becomes able to work with modern economic theory, for instance within the areas of within industrial organization, macroeconomics, international economics, labor economics, public economics and political economics.

In the process of the course the student will learn about
- Static games with complete information
- Static games with incomplete information
- Dynamic games with complete information
- Dynamic games with incomplete information
- Basic cooperative game theory.

For each of these classes of games, the student should know and understand the theory, and learn how to model and analyze some important economic issues within the respective game framework.

More specifically, the students should know the theory and be able to work with both normal and extensive form games. They should know, understand and be able to apply the concepts of dominant strategies, iterative elimination of dominant strategies, as well as mixed strategies. The students should know the central equilibrium concepts in non-cooperative game theory, such as Nash Equilibrium and further refinements: Subgame-Perfect Nash Equilibrium, Bayesian Nash Equilibrium, Perfect Bayesian Equilibrium. They should understand why these concepts are central and when they are used, and be able to apply the relevant equilibrium and solution concepts. Furthermore, the students should acquire knowledge about a number of special games and the particular issues associated with them, such as repeated games (including infinitely repeated games), auctions and signaling games.

The students should also understand and be able to apply the solution concepts of cooperative game theory, such as the core and the Shapley value. Furthermore, the students should also learn the basics of bargaining theory.

To obtain a top mark in the course the student must be able excel in all of the areas listed above.

Microeconomics C, Final Exam, January 2011
Suggested Solutions

January 21, 2011

1. (a) There are two pure strategy NE:

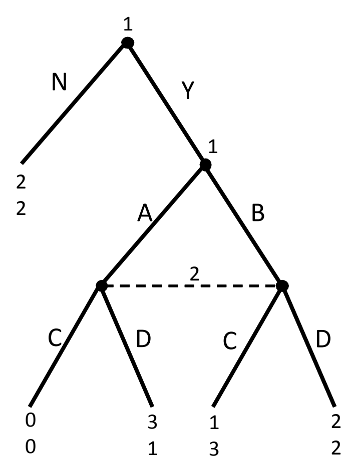
$$(T, L) \text{ and } (B, R).$$

To find the mixed strategy Nash equilibria, let p denote the probability that player 1 plays T and let q denote the probability that player 2 plays L. First note that there does not exist a (non-pure) MNE with $p > 0$ because then player 2's best response is L ($q = 1$) and player 1's best response to L is T ($p = 1$). Therefore let $p = 0$, i.e., player 1 plays B with certainty. Then all $q \in [0, 1]$ is a best response for player 2. But $p = 0$ (B) is only a best response to q if $q \leq \frac{1}{2}$. So the set of non-pure MNE can be written:

$$\{(p, q)|p = 0,\ 0 < q \leq \frac{1}{2}\}.$$

(This answer can also be found by plotting the best response correspondences of the two players in a (p, q)-diagram).

(b) i. The game tree can be drawn as follows (note that it is also perfectly fine to have player 2 moving first in the stage 2 game):



This is a game of imperfect information, there is one information set that consists of two decision nodes (because the stage two game is simultaneous).

ii. There is one subgame (excluding the game itself) - it starts after player 1 has chosen Y. The set of stategies for player 1 is

$$\{YA, YB, NA, NB\}.$$

The set of stategies for player 2 is

$$\{C, D\}.$$

iii. There are two pure strategy SPNE:

$$(YA, D) \text{ and } (NB, C).$$

(first find a pure strategy NE of the stage two game, then find player 1's optimal action in stage 1 given the NE in stage two).

iv. The normal form of the game is given by the following matrix:

	C	D
YA	$0,0$	$3,1$
YB	$1,3$	$2,2$
NA	$2,2$	$2,2$
NB	$2,2$	$2,2$

The pure strategy NE are:

$$(NA, C),\ (NB, C),\ (YA, D).$$

Comparing with the set of SPNE we see that one of the NE, (NA, C), is not subgame perfect. This is because it involves non-NE play in stage two - (A, C) is not a NE in the stage two game.

2. (a) Player i solves

$$\max_{y_i \geq 0} y_1 y_2 - C_i(y_i).$$

Best response functions (derived from the first order conditions):

$$y_1 = \sqrt{y_2} \text{ and } y_2 = \frac{1}{2} y_1.$$

NE with positive efforts:

$$(y_1^{NE}, y_2^{NE}) = (\frac{1}{2}, \frac{1}{4}).$$

(Note that $(0, 0)$ is also a NE).

(b) Player 1 solves (use the best response fct of player 2 from (a))

$$\max_{y_1 \geq 0} y_1(\frac{1}{2}y_1) - \frac{1}{3}(y_1)^3.$$

By the first order condition it follows that player 1 will choose

$$y_1 = 1.$$

And then player 2 will choose (use the best response from (a))

$$y_2 = \frac{1}{2}y_1 = \frac{1}{2}.$$

So the outcome in the SPNE is:

$$(y_1, y_2) = (1, \frac{1}{2}).$$

Formally, the SPNE is

$$(1, \frac{1}{2}y_1).$$

(c) In the NE from (a):

$$(U_1, U_2) = (\frac{1}{12}, \frac{1}{16}).$$

In the SPNE from (b):

$$(U_1, U_2) = (\frac{1}{6}, \frac{1}{4}).$$

Both players are better off in the dynamic game from (b). In the dynamic game player 1 can commit to an effort. If he commits to an effort that is higher than y_1^{NE} this will increase player 2's marginal benefit of his own effort and thus he will also choose $y_2 > y_2^{NE}$. This will make both players better off and thus they will be better off in the dynamic game.

(d) The social optimum is the solution to:

$$\max_{y_1, y_2 \geq 0} 2y_1y_2 - \frac{1}{3}(y_1)^3 - (y_2)^2.$$

By the first order conditions we get the following solution:

$$(y_1^{SO}, y_2^{SO}) = (2, 2).$$

Utilities at the social optimum:

$$(U_1, U_2) = (\frac{4}{3}, 0).$$

(e) Suppose we had a SPNE in which the outcome in each stage is $(y_1^{SO}, y_2^{SO}) = (2,2)$. Then the sum of discounted utilities for player 2 would be $\frac{1}{1-\delta}0 = 0$. If player 2 instead played the one-shot best response to $y_1^{SO} = 2$ in stage 1 (which is $y_2 = 1$) and then 0 in all later stages, his sum of discounted utilities would be $(2 \cdot 1 - 1^2) + \frac{\delta}{1-\delta}0 = 1 > 0$ (no matter what player 1 does in later stages). Thus we cannot have a SPNE in which the outcome in each stage is (y_1^{SO}, y_2^{SO}) because player 2 would have a profitable deviation.

3. (Note that the game considered in (a)-(c) is very similar to the game in Gibbons, figure 4.1.1, p. 176)

(a) Since there are no subgames in the game, the set of NE and the set of SPNE are identical. The normal form of the game is

	l	r
L	$0,0$	$4,3$
M	$1,1$	$3,2$
R	$2,4$	$2,4$

The set of pure strategy NE/SPNE is

$$\{(R,l),(L,r)\}.$$

(b) Let p be player 2's belief about the probability that player 1 have played L (given that player 2's info set is reached). For any $p \in [0,1]$ it is optimal for player 2 to play r. And given this strategy of player 2 it is optimal for player 1 to choose L. This means that (by Bayesian updating) we must have $p = 1$. Thus the only PBE is

$$[L, r, p = 1].$$

(c) In a PBE player 2 has to choose r if his information set is reached because r is optimal for any belief he might have. So player 2 cannot make player 1 play R by threatening to play l if his information set is reached (as he could if we used NE/SPNE). Since this threat is clearly "non-credible", PBE is the more appropriate equilibrium concept.

(d) The only separating PBE is

$$[(R,L),(d,u), p = 0, q = 1].$$

I.e., t_1 plays R, t_2 plays L, the receiver plays d after observing the message L and u after observing the message R.

(If the strategy of the sender is (L,R) then the strategy of the receiver must be (u,d). And then it is a profitable deviation for t_2 to send the message L instead of R.).